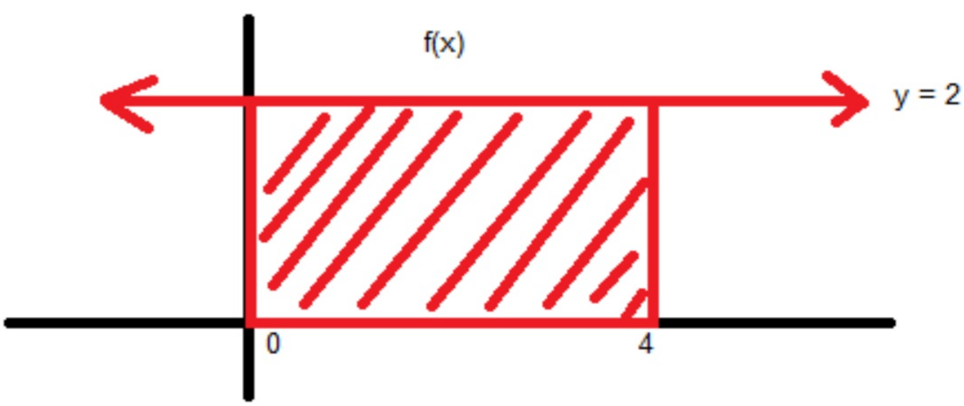
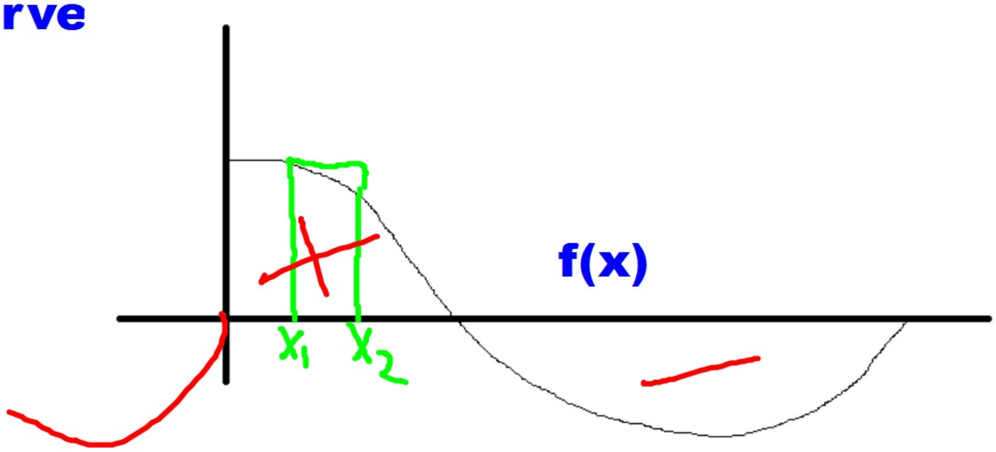
Section 5.2
Definite Integrals

Find the area under the curve.



Riemann Sums

Using rectangles to approximate area under the curve



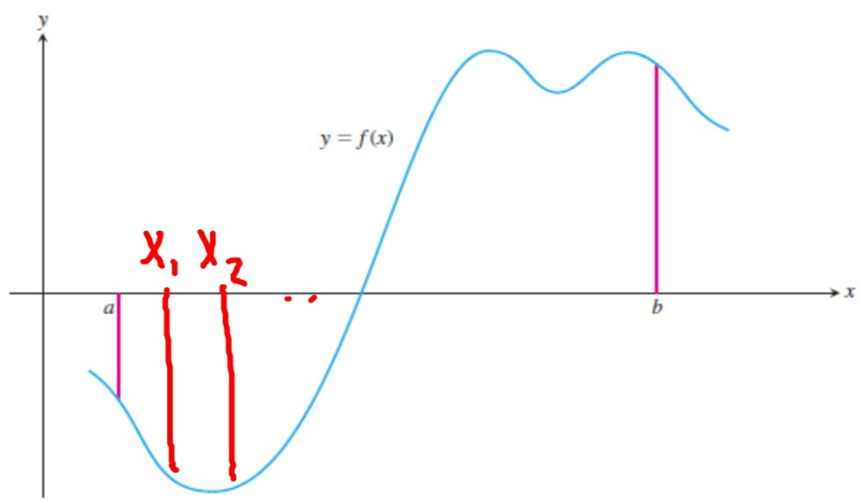
$\sum_{k=1}^{6} f(c_k) \cdot \Delta x_k$ **is approximately the area under curve ("net") area**

1.) Partition the interval [a,b] into n subintervals

2.) We know that $a < x_1 < x_2 < \dots < x_{n-1} < b$.

3.) Partition of [a,b] is

$$P = \{x_0, x_1, x_2, \dots , x_n\}$$



The Definite Integral of a Continuous Function on [a,b]

$$\lim_{n \to \infty} \sum_{k=1}^{n} f(c_k)\, \Delta x_k$$

height. width.

As n increases, the approximation gets better!!!

Upper limit of integration

The function is the **integrand.**

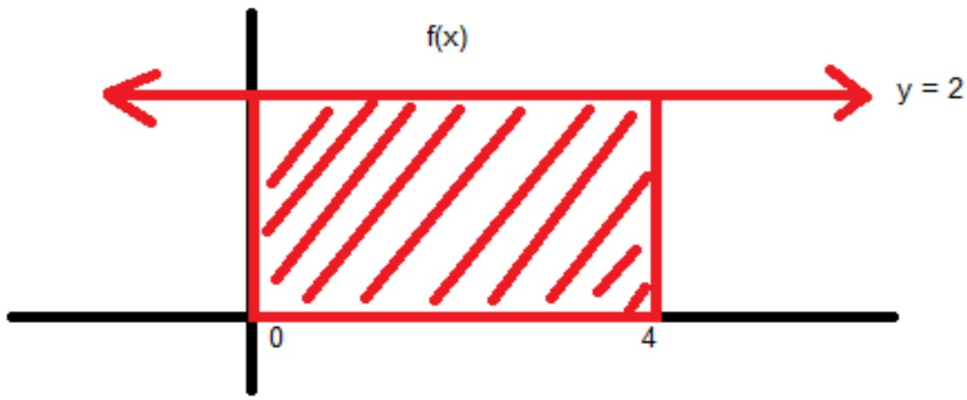
Integral sign

x is the **variable of integration.**

$$\underbrace{\int_a^b f(x)\,dx}_{\text{Integral of } f \text{ from } a \text{ to } b}$$

Lower limit of integration

When you find the value of the integral, you have **evaluated the integral.**



$$\int_0^4 2\,dx = 8$$

Definite Integral

"NET" area

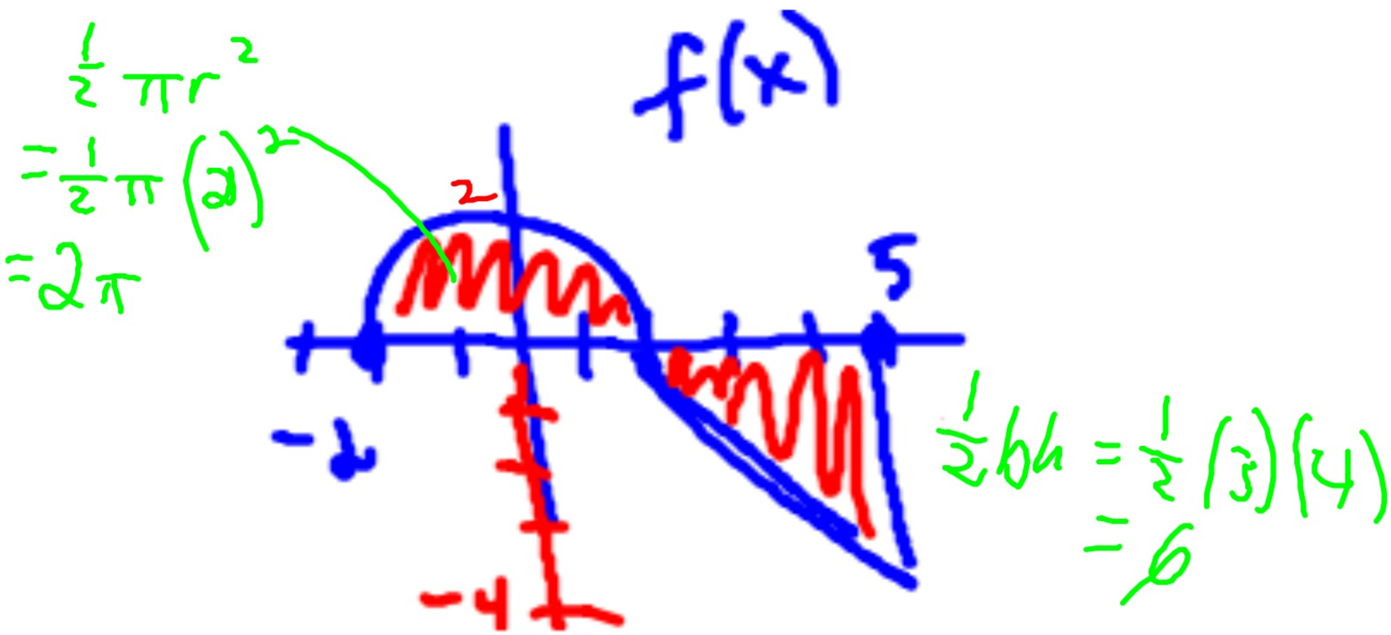
Area Under a Curve (as a Definite Integral)

$$A = \int_{a}^{b} f(x)\,dx$$

$$\text{area} = -\int_a^b f(x)\,dx \text{ when } f(x) \leq 0$$

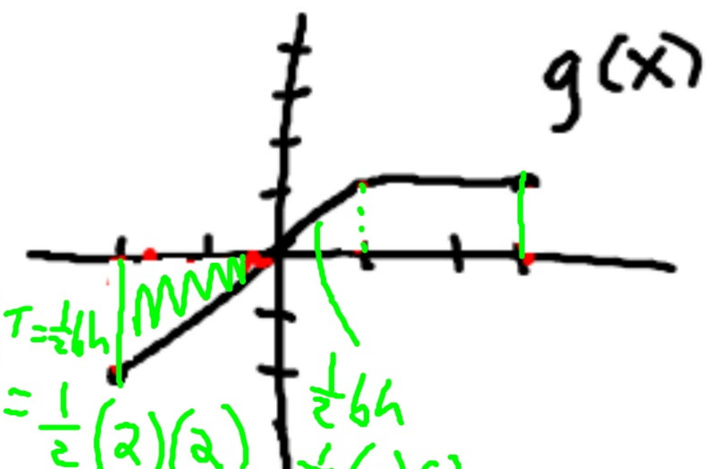
Can have a negative area!!!

Find $\int_{-2}^{5} f(x)\,dx = 2\pi - 6$



Find $\int_{-2}^{3} g(x)\,dx = -2 + \frac{1}{2} + 2$

$= \frac{1}{2}$



A of T $= \frac{1}{2}bh$

$= \frac{1}{2}(2)(2)$

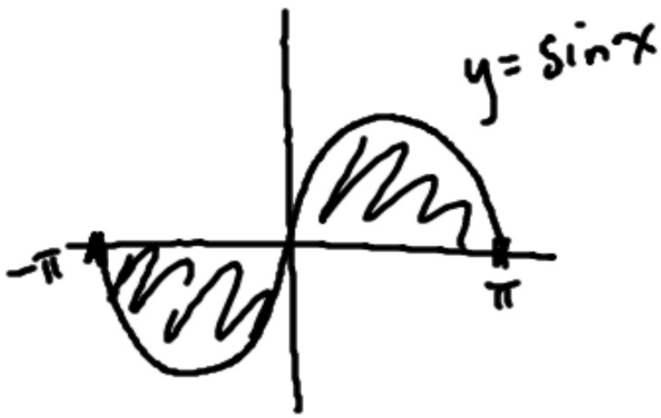
$= 2$

$\frac{1}{2}bh$

$\frac{1}{2}(1)(1)$

$\frac{1}{2}$

Find $\displaystyle\int_{-\pi}^{\pi} \sin x \, dx = 0$

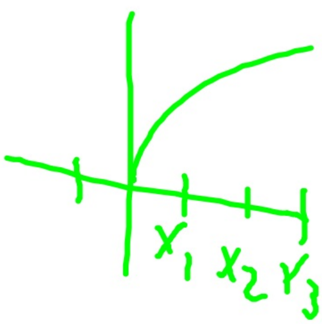


Find $\displaystyle\int_{0}^{\pi} \sin x \, dx =$

The interval [-1,3] is partitioned into n subintervals of equal length $\Delta x = 4/n$. Let m_k denote the midpoint of the k^{th} subinterval. Express the limit

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left(3(m_k)^2 - 2m_k + 5\right) \Delta x$$

as an integral.



$$\int_{-1}^{3} \left(3x^2 - 2x + 5\right) dx$$

Evaluate the integral

$$\int_{-2}^{2} \sqrt{4 - x^2}\ dx = 2\pi$$

$$A = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(2)^2 = 2\pi$$

PLORATION 1 Finding Integrals by Signed Areas

It is a fact (which we will revisit) that $\int_0^{\pi} \sin x \, dx = 2$ (Figure 5.20). With that information, what you know about integrals and areas, what you know about graphing curves, and sometimes a bit of intuition, determine the values of the following integrals. Give as convincing an argument as you can for each value, based on the graph of the function.

1. $\displaystyle\int_{\pi}^{2\pi} \sin x \, dx$
2. $\displaystyle\int_{0}^{2\pi} \sin x \, dx$
3. $\displaystyle\int_{0}^{\pi/2} \sin x \, dx$
4. $\displaystyle\int_{0}^{\pi} (2 + \sin x) \, dx$
5. $\displaystyle\int_{0}^{\pi} 2 \sin x \, dx$
6. $\displaystyle\int_{2}^{\pi+2} \sin (x - 2) \, dx$
7. $\displaystyle\int_{-\pi}^{\pi} \sin u \, du$
8. $\displaystyle\int_{0}^{2\pi} \sin (x/2) \, dx$
9. $\displaystyle\int_{0}^{\pi} \cos x \, dx$
10. Suppose k is *any* positive number. Make a conjecture about $\int_{-k}^{k} \sin x \, dx$ and support your conjecture.

The Integral of a Constant

If $f(x) = c$, where c is a constant, on the interval $[a,b]$, then

$$\int_a^b f(x)\,dx = \int_a^b c\,dx = c\,(b - a)$$

A train moves along a track at a steady 75 miles per hour from 7:00 am to 9:00 am. Express its total distance traveled as an integral.

$$\int_7^9 75\,dx = 75(9-7)$$

$$= 150$$

Integrals on a calculator

$$\int_a^b f(x)\,dx = \text{NINT}\,(f(x), x, a, b)$$

Evaluate the following integrals numerically.

a.) $\displaystyle\int_{-1}^{2} x \sin x \, dx$

b.) $\displaystyle\int_{0}^{1} \frac{4}{1 + x^2} \, dx$

c.) $\displaystyle\int_{0}^{5} e^{-x^2} \, dx$

All continuous functions are integrable...

Some functions with discontinuities are also integrable...

Example

Find $$\int_{-1}^{2} \frac{|x|}{x}\,dx$$

1.) Explain why the function

$$f(x) = \frac{x^2 - 4}{x - 2}$$

is not continuous on [0,3].

2.) Use area to show that

$$\int_0^3 \frac{x^2 - 4}{x - 2}\,dx = 10.5$$